The relationship between emitted frequency f and the detected frequency f', when either, or both, the source and detector/observer are moving is shown by:

$$f' = f\frac{v \pm v_D}{v \pm v_s}$$

Where v=speed of sound through air, v_D=detector's speed relative to the air, and v_S=source's speed relative to the air.

Any time the detector (D) is stationary, the wavefronts move a distance vt Velocity*time=distance, or in this case, the speed of sound through the air multiplied by time elapsed is equal to the distance the sound travels.

The number of wavelengths in this period of time is equal to the distance travelled by the sound, divided by the wavelength. This is also equal to the number of wavelengths intercepted by D.
Wavelength= λ
Distance the wave travels in time t is equal to vt
Therefore, the number of wavelengths that occur in the amount of time elapsed is given by:

$$\text{number of wavelengths} = \frac{vt}{\lambda}$$

This is also equal to the number of wavelengths intercepted by D. Therefore, the rate that D intercepts wavelengths (number of wavelengths that pass a certain point in a certain amount of time, otherwise known as frequency) can be written as follows:

$$f = \frac{vt/\lambda}{t} = \frac{v}{\lambda}$$

In this case, D (the detector of the sound) and S (the source of the sound) are both stationary. As a result, there is no Doppler effect and the frequency detected is equal to the frequency emitted by S.

In this next case, D is traveling opposite the direction that the sound is traveling, and the source of the sound is stationary. D is moving toward S, in the opposite direction of the wavefront velocity. D passes wavelengths faster than it would when stationary. This is because the speed the sound travels isn't changing, however, D is now approaching the waves at a given velocity. The waves then reach the detector (D) faster, because as time passes, and D is traveling at a certain velocity, the distance the wave has to travel to get to D decreases. As a result of passing wavelengths and reaching each crest and trough faster than normal, the period decreases. The period being the time that elapses between two consecutive peaks or troughs in the wave.

D moves opposite in the direction the wave is traveling, at a certain velocity, over a certain amount of time. Therefore, D moves a distance equal to $v_D t$.

The wavefronts movement (and distance traveled) relative to D (in this case) is $vt + v_D t$. The wavefront and D are moving a certain distance toward one another, and the relative distance moved by the wavefront must be the distance covered by the wave, in addition to the distance travelled by the detector in that time interval. That is how we get to the formula $vt + v_D t$.

The number of wavelengths intercepted by D can be represented by:

$$\frac{vt + v_D t}{\lambda}$$

The rate the wavelengths are intercepted, the number of wavelengths that pass D in a certain amount of time, is going to be the frequency detected by D (f'). The formula would then be as follows:

$$f' = \frac{vt + v_D t}{\lambda t} = \frac{v + v_D}{\lambda}$$

$f = \frac{v}{\lambda}$ therefore, $\lambda = \frac{v}{f}$ and then the equation above can be rearranged as follows:

$$f' = \frac{v + v_D}{v/f} = f\frac{v + v_D}{v}$$

Because v is in the numerator and denominator, one may notice that f' will always be greater than f unless v_D=0, which only occurs when D is stationary, and then the frequency detected would be the same as the frequency emitted.

If $v_D = 0$ plugging into the formula, one would get: $f' = f\frac{v + 0}{v} = f\frac{v}{v} = f$

therefore, when $v_D = 0, f' = f$

In this next case, D is moving away from the source, in the direction the wavefront is traveling. The relative distance traveled by the wave will equal $vt - v_D t$. Therefore,

$$f' = f\frac{v - v_D}{v}$$

In this case, f' must be less than f, unless v_D=0.

The formulas for the frequency of the sound detected (when approaching or moving away from the source) can then be summarized as follows:

$$f' = f\frac{v \pm v_D}{v}$$

If the source is moving, the wavelength of sound waves it emits changes, which then changes the frequency detected by D.

T=$1/f$ is the time that elapses between the emission of two consecutive waves (W_1 and W_2)

The distance traveled by W_1 is equal to vT, and the distance traveled by the source over T is v_sT. After, T, the second wave is emitted. Therefore, the distance between the two waves, the wavelength, is equal to vT— v_sT.

So, if λ' is the wavelength of those waves, then:

$$f' = \frac{v}{\lambda'} = \frac{v}{vT - v_s T} = \frac{v}{{v}/{f} - {v_s}/{f}} = f\frac{v}{v - v_s}$$

f' must be greater than f unless v_s=0

In the opposite direction that S is traveling, the wavelength will be $vT + v_s T$

Then frequency will be detected as:

$$f' = f\frac{v}{v + v_s}$$

The following can summarize the two detected frequency formulas used for movement of S:

$$f' = f\frac{v}{v \pm v_s}$$

By combining this formula with the formula summarizing detected frequency when D moves, one gets:

$$f' = f\frac{v \pm v_D}{v \pm v_s}$$

This is because if D isn't moving one would still have

$$f' = f\frac{v}{v \pm v_s}$$

If S weren't moving, one would still have

$$f' = f\frac{v \pm v_D}{v}$$